# Introduction

Gärdenfors distinguishes between two main trends in semantic theories: on the one hand, a *realist* approach, according to which the meaning of words is derived from the external world, and on the other, a *cognitive* approach, according to which meaning is derived from projects and dreams.[[1]](#footnote-1)

The Swedish philosopher places his theory of conceptual spaces more within this second orientation, since it models the dynamics of semantic representations without direct reference to the outside world: meaning is only in the minds of the speakers. What counts above all is to account for the *effectiveness of* communication between speakers and cognitive operations such as *categorisation* and *induction,* rather than a correspondence between language and the external world. The reference to the outside world is not abandoned, but it is used indirectly.[[2]](#footnote-2)

## What is a conceptual space?

Each conceptual space is a geometric representation of certain natural language concepts. It is made up of dimensions, each of which represents a *quality* such as colour, pitch, temperature, etc.[[3]](#footnote-3) Gärdenfors offers the following general description :

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| --- |
| The central idea is that the meanings we use in communication can be described as organised in abstract spatial structures expressed in terms of dimensions, distances, regions and other geometric notions.[[4]](#footnote-4) |

Each object is characterised by values on these dimensions, each value representing a respective quality of that object.

***1.1. Dimensions and domains***

A domain is a set of intrinsically linked and inseparable dimensions. A domain is *typically* a sensory modality.[[5]](#footnote-5) For example, colours are a domain: a visual stimulus always comprises the dimensions of *luminosity* and *saturation*, perceived jointly and inseparably. In the same way, a sound is invariably characterised by a pitch and an intensity.

* 1. ***Properties, concepts and instances***

In a conceptual space, a ***property is*** represented by a *region* within a single domain.[[6]](#footnote-6) A property corresponds to a quality or attribute, often expressed by an adjective. The *space* contained within this region corresponds not only to information about members already observed, but also about *potential* members not yet encountered. As it is defined on several dimensions of a domain, the extension of the region indicates all the possible combinations of values on these dimensions for the members of the corresponding category. For example, the Red property is understood as a *region* in the colour domain. These properties are said to be *graded* because they can take *continuous* values in the intervals defined by each dimension of the domain. More precisely, if is the colour domain, then for each dimension there exists an interval of values corresponding to the extension of red in that dimension.

In a conceptual space, each ***instance*** or ***copy*** of a property is represented by a *point* located within the region corresponding to this property. In everyday language, the expression "specific colour" can refer to either a *region* or an *exemplar*. To avoid this ambiguity and to distinguish between these two types of use, when a word refers to a *region*, we will capitalise it. For example, this difference appears in the sentence: "a shade *of* red is a *point* in the region *of* Red". This notation is used to distinguish *small regions* corresponding to *specific properties* from simple points representing *instances*. So, instead of writing that Carmine *belongs* in Red, we can use an *inclusion* relation, for example in the sentence: "Carmine is contained in Red" to express that the former is a small region contained in the latter. The distance between two points expresses the perceived similarity between them. The greater the distance, the less similar they are, and vice versa.

Unlike *properties*, ***concepts*** cover several domains. Each concept is *defined* by a combination of regions from different domains. These regions contain *all the* possible *instances* of the concept in their respective domains. For example, the concept 'apple' is represented by regions in four domains: colour (*e.g.* red, green, yellow), taste (*e.g.* varying from sweet to tart), shape (*e.g.* round, oval, cycloid), texture (*e.g.* crunchy, soft). More than just *qualifiers*, these regions *define* the concept of apple. As mentioned above, the regions on these domains are not identified with *binary predicates*: a singular apple is a *point* whose coordinates reflect the precise values of this apple in terms of colour, taste, shape and texture (*e.g.* a dark red apple, sweet, etc.). Such a decomposition works in principle for any concept, e.g. "bird", encompasses properties spread across different domains such as shape, size, colour, weight, etc.

Let's show that a concept can be *broken down* into *regions* in different domains, and how these domains can each be *broken down* into a small number of dimensions.

The region in conceptual space that represents a concept can be seen as a subset of the Cartesian product of different domains:

where each domain is generally defined in one, two or three dimensions and has a *finite* total area/volume[[7]](#footnote-7) . For convenience[[8]](#footnote-8) , each dimension of a domain is identified with an interval[[9]](#footnote-9) bounded in . An instance of the concept can be represented by a ***point*** :

where each represents a point in the domain and therefore corresponds to a vector of dimension according to the number of dimensions in the domain . In other words, if is an instance of then the component of is located in the region associated with the concept on the domain . This region is strictly included in . Formally, if the region associated with the concept in the -dimensional domain is denoted then :

Thus the representation of a *concept* is ***compositional***: all the specific instances of a concept are points located in each of its regions according to their attributes on each dimension.

It should be noted, however, that the compositionality on which the theory of conceptual spaces is based is *weaker* than linguistic compositionality, which postulates that the semantics of a concept can be broken down into *known* linguistic units. Indeed, in general, *no known adjective* corresponds precisely to the region corresponding to a concept in a given domain. Despite this, it is often possible to approximate the region corresponding to this concept by *a union of small regions*. To illustrate this, let's take as an example the region corresponding to the concept "lemon" in the colour domain, denoted by . This region seems to be approximated by :

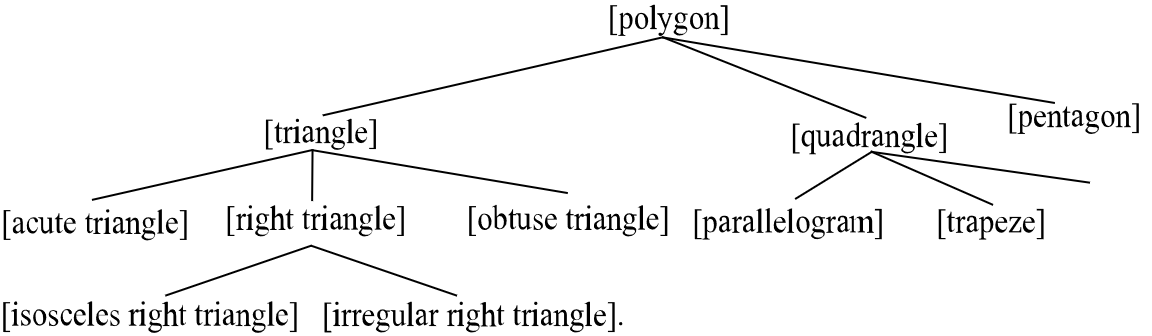
However, the more nuances there are in the regions that are never instantiated by any lemon, and the more nuances there are in that are not in , the more imperfect the approximation.[[10]](#footnote-10) A measure of this imperfection could be given by :

where the numerator represents the shades of colour that are common to the region and and the and where the denominator represents the total number of *distinctive shades*: those found *only* in and those found *only* in . If the denominator is non-zero, this shows that the union of the two regions associated with the adjectives "yellow" and "light green" is still an *approximation of* the concept of "lemon" in the colour domain.

## The equivalence between tree and spatial representations

In his *Organon,* Aristotle defines *genus* (γένος) as a *predicate* common to several things having *specific differences* between them.[[11]](#footnote-11) Thus *genus* is an essential attribute shared by several *species* (εἶδος). For example, "animal" is the genus common to the human species, the species of horses, birds, etc. Each species is defined by adding a specific difference to the genus. In this context, the species "lions" can be defined as *felines* (genus) with a *mane* (specific difference). The genus therefore has greater extension, because it includes several species, whereas the species requires more conditions in its definition, because it contains additional determinations compared with the genus.[[12]](#footnote-12)

The relationship "X and Y are *two kinds* of *Z*" (or equivalently "Z is the *kind of* X and Y") can be represented in two distinct ways: either ***aborescently***, where X and Y are two lower nodes connected to the upper node Z, or ***spatially***, where X and Y are *enclosed* within Z as if in a box. The properties that define Z are also assigned to X and Y. Interpreted in a tree structure, this means that lower nodes *inherit* the properties of higher *nodes.* Interpreted spatially, this means that a property defining a set is *passed on* to its subsets. So, the *higher* a node is in the tree structure, the more *general* the properties that characterise it are, and vice versa. Similarly, the less restrictive and specific the conditions required to belong to Z, the broader the extension of Z will be, and vice versa. These principles seem to be particularly well exemplified in the taxonomies of living beings, or in those of mathematical objects. For example, Nicole and Arnauld (1662) proposed a hierarchical tree structure for a number of *geometric concepts*[[13]](#footnote-13) :



Each node in the tree represents a set of objects satisfying specific properties. Nicole and Arnault specify that the set of triangles "can be divided according to the sides, or according to the angles".[[14]](#footnote-14) The division according to angles is illustrated in the tree structure above, while the division according to sides would have produced three branches: isosceles, equilateral and scalene triangles. The division depends on the property being considered.

## Represent the semantics of ordinary concepts as regions : a challenge for the theory of conceptual spaces

As mentioned, the hierarchical relationships between these concepts can be represented either by a *tree structure* or *spatially*, where the extension of the concepts is represented by boxes enclosed within each other. In contrast to tree representation, spatialization makes it possible to quantify measures such as the *size of* the different boxes or the *distances* between them. A large part of research in the context of conceptual spaces wants to seize these advantages: it is not enough to write: "Concept A ⊂ Concept B", but we must try to determine the *size of* the extension of A in relation to that of B, to determine the *distance* separating the extensions of A and B. The ambition to determine *the size* and *distances* between the semantic extensions of concepts has met with some success for sensory categories and particularly for *colours*. For other concepts in ordinary language, although there are many techniques for measuring the distance between two words (e.g. between '*cat'* and '*dog'*), it generally seems very difficult to estimate *the size of* the two regions corresponding to the semantics of these two concepts.

For about a decade, some research into conceptual spaces has been inspired by *word emebedding models*, which represent words in the form of *vectors*[[15]](#footnote-15), i.e. ***points*** in spaces with several hundred ***non-interpretable*** dimensions. These models allow each word to be represented by a single vector, which encodes its position relative to all the other words.[[16]](#footnote-16) This approach seems to us to be in tension with the theory of conceptual spaces in at least two respects.

Firstly, the theory of conceptual spaces represents concepts as ***regions*** that delimit the set of instances that satisfy the concept's membership conditions. Representation with points does not account as well for the variability and flexibility of the semantics of the concepts concerned. Secondly, the individual dimensions that make up word-vectors in word embedding models are generally very difficult to interpret[[17]](#footnote-17) , whereas in conceptual spaces, each dimension represents an ***interpretable*** quality[[18]](#footnote-18) (*e.g.* for *colours:* the degree of brightness, hue, saturation (1.1), for *taste:* the degree of acidity, sweetness[[19]](#footnote-19) , etc.). There are many techniques that can be used to *reconcile emebedding models* of *words* with conceptual spaces. Before detailing them, the motivations are given in the following paragraph.

Is the difficulty in interpreting word vectors in these word embedding models due to the fact that they have several hundred dimensions? This question leads to another: are only low-dimensional spaces interpretable?

In reality, in the theory of conceptual spaces, it is only the *domains that are* of low dimensionality. For example, the semantics of an adjective that refers to *a single* sensory domain[[20]](#footnote-20) (such as "blue", "bitter", "sharp") is generally represented in a three-dimensional space. However, for words that do not refer to a single sensory domain, visualisation is only possible for each semantic component considered in isolation.

A frequently used example is the concept of *apple*. The semantics of this concept are represented in the domains of colour, shape, taste, texture and nutritional and biological specifications[[21]](#footnote-21) . The regions in these different domains can all be visualised: for example, the three-dimensional region containing all the possible colours for apples can be represented in colour space. However, if the *global semantics* of the concept of apple involves all these domains simultaneously, it becomes difficult to visualise in a low-dimensional space where each dimension is interpretable. This conceptual space for the concept of an apple is *"hand-crafted"*[[22]](#footnote-22) , i.e. the properties of the concept are determined a priori. In general, "hand-made" spaces have the merit of defining the concept in an *interpretable* and *intuitive* way*,* but have the disadvantage of having no predictive power, since they do not define each property of the concept *quantitatively*. This is one of the reasons why, today, some research attempts to obtain conceptual spaces by processing data. So let's explain the steps we need to take to do this.

## How to build a conceptual space?

***Data collection.*** Before constructing a conceptual space, it is first necessary to obtain *data* on the *similarity of* the elements to be represented in it.[[23]](#footnote-23) This data collection can use both experimental and computational methods: humans can be asked to estimate the similarity between all possible pairs with these elements[[24]](#footnote-24) . Similar data can be obtained using *word embedding* models and/or large language models (LLMs)[[25]](#footnote-25) . We can also manually run *co-occurrence statistics* between certain words on targeted text corpora to estimate their semantic proximity.[[26]](#footnote-26) The data is arranged in a *similarity matrix*.[[27]](#footnote-27) This matrix reflects the *similarity judgements* between these elements: each cell of the matrix contains the similarity of with . The raw data in such a matrix has large dimensions.

***Dimensionality reduction.*** Although there issome diversity in the *sources* and *protocols that* can be used to obtain this similarity data, the *same* dimensionality reduction *technique* is used almost systematically in the literature to construct conceptual spaces from this data. This technique is known as *Multidimensional* Scaling (MDS).[[28]](#footnote-28) MDS makes it possible to project these elements into a lower-dimensional space, while preserving the initial similarity relationships as far as possible.[[29]](#footnote-29) More specifically, it involves minimising a cost function called *Stress*, which measures the difference between the initial similarities and the distances in the projected space.[[30]](#footnote-30) The quality of a spatial representation obtained by an MDS can be visualised in a graph called a *Shepard diagram*. More precisely, a Shepard diagram represents the relationship between the initial similarities in the matrix and the distances obtained in the space after application of the MDS. The similarity relationships of the initial data are plotted on the x-axis, while the distances between pairs of elements are plotted on the y-axis. Ideally, if the distances in the projected space correspond perfectly to the initial similarities, the points in a Shepard diagram should lie exactly on a monotonically decreasing line[[31]](#footnote-31) . Conversely, the less faithful the space is to the initial dissimilarities, the further these points will deviate from this straight line. In general, low-dimensional spaces are more interpretable, but risk being less faithful to the data. Conversely, high-dimensional spaces are more faithful to the data, but less interpretable.[[32]](#footnote-32) Shepard diagrams help to find the best compromise between these two constraints.

Applying an MDS to a matrix containing similarity relations produces a *similarity space,* but not yet a *conceptual space*. A conceptual space is a *special case* of a similarity space: it has *low dimensionality,* and distance is identified with dissimilarity. To enrich a similarity space in such a way as to transform it into a conceptual space, the following two additions are required:

(***1***) In a similarity space, dimensions may not be interpretable, whereas in a conceptual space, dimensions have ***meaning***.

(***2***) In a similarity space, each concept and property is represented by a *point, whereas* in a conceptual space they are represented by ***regions.***

To obtain a true conceptual space, it is therefore necessary to enrich the expressiveness of the similarity space by *interpreting* its dimensions and constructing *regions* within it. It seems to us that the need for *interpretability* sometimes comes into tension with the need for model *prediction performance.* Let us now present the case of colours in which these two constraints have been reconciled, and then point out the limits of applications to other concepts where reconciliation seems more difficult.

The conceptual space of colours is at once *interpretable*, partitioned into *regions* and allows human judgements about colours to be accurately predicted[[33]](#footnote-33). According to Douven and Gärdenfors (2019), this application is a *success, as the regions* corresponding to the categories of colours seem to follow certain optimality constraints (*cf.* [1.4.4](#_Categorisation_in_prototype)). The application of the theory of conceptual spaces to colours is in line with the work of Shepard (1974), who already sought to place colours in an *interpretable* "psychological space". The application of the theory of conceptual spaces to colours thus inherits Shepard's ambition to extract *general laws* from empirical data, where the variables are interpreted by objects of cognitive psychology, but whose relationships are mathematical. A key example is that of the similarity judgement as a function. From data on similarity judgements between colours, Shepard infers a *function* whose idea is that the similarity between two colours is a decreasing exponential function of their distance in the "psychological space of colours". Shepard's law states that the degree of similarity between and is given by a function defined as follows:

where is the distance between and in psychological space, and is a sensitivity parameter that controls the *rate* at which similarity decreases as a function of distance. Shepard's ambition is to use such a function for stimuli of different natures (visual, auditory, etc.) and for different species (humans, monkeys, birds, etc.).[[34]](#footnote-34) This function is widely used in research as part of the theory of conceptual spaces[[35]](#footnote-35) . One of the ideas that Gärdenfors takes from Shepard is that the *further apart* two points are in the similarity space, the less likely it is that they belong to the *same* category, and conversely, the closer they are, the more likely it is that there is a category whose extension contains them both.[[36]](#footnote-36) This hypothesis is crucial because it suggests that points belonging to the same category tend to appear as *clusters* in a similarity space. If a specific category groups together elements that are homogeneous and similar to each other, they should take the form of a dense cluster that is separable from the rest of the points in the similarity space.[[37]](#footnote-37) Conversely, the more heterogeneous a category, the more *diffuse* the cluster of points representing its elements will be, and the more *spread out the* points will be in the similarity space. By combining this hypothesis with the prototype theory[[38]](#footnote-38) , Gärdenfors was able to propose a simple and elegant geometric technique for constructing *regions* from *points* (cf. 1.4.4).

One of the advantages of Shepard's hypothesis is that it can be *experimentally tested* with any items. The first step is to collect similarity judgements between a list of predefined words, for example "oak", "poplar", "maple", "apricot", "banana", "cherry", then apply an MDS to obtain a similarity space, and finally check whether the points corresponding to words in the same category, in this case *fruit trees*, form a dense and small cluster. This is not the only possible category; other characteristics can be used to account for the proximity between points in this similarity space.[[39]](#footnote-39)

To make this more concrete, let's give another example: we can determine a list of animals, which are linked to characteristics such as their respective habitats (terrestrial, aquatic, aerial) and their sizes (small, medium, large). The data derived from similarity judgements about these animals can be *projected* into a similarity space by applying an MDS. This space can then be interpreted on the *basis of* the list of characteristics initially established. This is precisely what Douven et al (2023) propose, with the following similarity space:

Une image contenant texte, capture d’écran, nombre, diagramme

Description générée automatiquement

Fig. A. Similarity space containing 20 mammals. Extract from Douven et al (2023)

The *y-axis* seems to correlate with the ferocity of the twenty mammals[[40]](#footnote-40) and the *x-axis* seems to correlate with their size. We can try to make sense of the different clusters, identifying the bottom one as corresponding to *small animals*, the one on the left as corresponding to *herbivores*, and the top one to *carnivores*.[[41]](#footnote-41)

In this dissertation, we explore the question of whether conceptual spaces are suitable *only* for sensory categories, and difficult to apply to categories whose meaning encompasses a wider variety of domains. In fact, the use of *domains, i*.e. sets of indissociable dimensions, seems *prima facie* particularly suitable for sensory categories. On the one hand, Gärdenfors (2014) argues that the dimensions of conceptual spaces are closely related to what is produced by sensory receptors[[42]](#footnote-42) , on the other hand, he writes that there are also abstract, non-sensory qualitative dimensions[[43]](#footnote-43) . Research has often focused on the modality of *colours* defined by the dimensions of hue, intensity and luminosity. Other research has applied it to the category of *sound*, using dimensions such as pitch and intensity.[[44]](#footnote-44) Still others have applied it to the *taste* modality with dimensions such as sweet, sour, bitter and salty.[[45]](#footnote-45)

However, to our knowledge, there is currently no model that can represent the semantics of natural language concepts by combining (i) *interpretability* of dimensions, (ii) representation of concepts as *regions* and (iii) predictive performance.

1. Gärdenfors (2000, sec. 5.1.1), (2014, sec. 1.2). [↑](#footnote-ref-1)
2. If the conceptual space were completely out of touch with reality, it would probably not be viable. [↑](#footnote-ref-2)
3. Gardenfors (2015, sec. 3.5): "Examples of such dimensions are: color, pitch, temperature, weight". [↑](#footnote-ref-3)
4. Gärdenfors (2014, sect. 2.1): "A central idea is that the meanings that we use in communication can be described as organized in abstract spatial structures that are expressed in terms of dimensions, distances, regions, and other geometric notions. In addition, I also use some notions from vector algebra". [↑](#footnote-ref-4)
5. Bechberger (2023, p.14), "quality dimensions are (...) typically assumed to be based on perception". [↑](#footnote-ref-5)
6. Gardenfors (2000, sec. 3.5); (2015, p.4): "A natural property is a convex region of a domain in a conceptual space".

   Gardenfors (2014, sec. 2.2): "I use the notion of a property to denote information related to a single domain". [↑](#footnote-ref-6)
7. For sensory domains such as vision and hearing, this finiteness is particularly intuitive. This means that for a dimension such as pitch ("high/low"), there are limits beyond which we do not perceive sounds as higher or lower. The same applies to vision, where similar limits exist for colours or light intensities... [↑](#footnote-ref-7)
8. In practice, a point is never *infinitely* like in but it is, in conceptual space, a region small enough to be considered as such (cf. 2.5.2). [↑](#footnote-ref-8)
9. Regions are often approximated by Cartesian products of intervals. For example, the product can be visualised as a rectangular region whose opposite vertices have coordinates (a, c) and (b, d). The product of *three* intervals is visualised as a right block. Visualisations in Bolt et al (2019, p.167-172). [↑](#footnote-ref-9)
10. Another flaw in this approximation could be the *uneven distribution of* lemons between these two colours (for example, if they are more often yellow than green)*.* [↑](#footnote-ref-10)
11. Aristotle, Organon, *Topiques* I, 5, 102a31-32. [↑](#footnote-ref-11)
12. Aristotle, Organon, *Topiques* IV, 1, 121a10-14. [↑](#footnote-ref-12)
13. Nicole & Arnault (1992, p.311) use a brace sign to note the partition of a set into subsets. They describe these relationships using the notions of genus and species, for example: "the quadrilateral, which is a genus with respect to the parallelogram and the trapezoid, is a species with respect to the figure" (1992, p. 53). [↑](#footnote-ref-13)
14. Ibid (1992, p.311). [↑](#footnote-ref-14)
15. More details in section (2.3.2). These vectors are represented in spaces with dimensionality between 100 and 1000 (Mikolov et al. 2013). [↑](#footnote-ref-15)
16. Derrac & Schockaert (2015, p.69): "Most approaches represent natural language terms as points or vectors". [↑](#footnote-ref-16)
17. Bechberger (2023, p. 271): "the neural networks is quite opaque and cannot be easily analysed or interpreted by human experts". [↑](#footnote-ref-17)
18. Bechberger (2023, p. 271): "Interpretable dimensions, however, are a corner stone of the conceptual spaces framework". [↑](#footnote-ref-18)
19. Other dimensions can also be taken into account in the taste domain (Bolt et al. 2019, p.168). Although degree of acidity is generally a separate dimension from degree of sweetness and they are separated from the colour domain, Bechberger (2023, pp. 89-97) formalises the correlation between the green colour of an apple and its acidity, as well as between the red colour of an apple and its sweetness. [↑](#footnote-ref-19)
20. When they are conceived as *properties*, adjectives are *restricted* to *a single* domain, in which their *literal meaning* is found. The possible *connotations of* the adjective are not captured in this single domain. For example, the fact that the adjective "blue" connotes the sea, the sky, or calm implies leaving the domain of colours. Since *properties* can only be defined in *one* domain, the representation of connotations only seems possible for *concepts*. [↑](#footnote-ref-20)
21. Gardenfors (2000, sec. 4.2.1), Fiorini (2014), Bechberger (2023, p.87). [↑](#footnote-ref-21)
22. Bechberger (2023, p.46): "Handcrafting a conceptual space usually consists in manually defining the dimensions of the conceptual space based on the available sensors". [↑](#footnote-ref-22)
23. Douven et al (2017, p.690*)* [↑](#footnote-ref-23)
24. In practice, it is not always *necessary* for each participant to give similarity judgements. For example, to collect similarity data between 20 different mammals, Douven et al (2023) use a "spatial arrangement task" in which all the words are randomly arranged on the screen and the participants have to rearrange and reposition them so that the most similar words are close to each other. They found this process faster and less tedious for the participants.

    For similarity data between *colours*, see Borg and Groenen (2005, p.65), who give the similarity matrix between *colours* obtained by Ekman (1954), details in section ([1.4.1](#_Using_psychophysical_experiments)). Douven et al (2017, p.690), details in section ([1.5.2](#_1.5.2._An_experiment)) [↑](#footnote-ref-24)
25. Sections (2.3.2) to (2.3.5) below give more details on the methods that can be used. Moullec & Douven (2024) obtain quality similarity judgements between different mammals using, in particular, Word2vec, FastText and GPT4. [↑](#footnote-ref-25)
26. Bendifallah et al. (2023): "(...) we derive similarities from how often our participants group figures together in some group, which we can represent in a co-occurrence matrix".

    Derrac & Schockaert (2015): "The required similarity degrees are often obtained from (...) the co-occurrence".

    One technique that can be used to quantify co-occurrence between two words is PMI, *cf*.[Appendix A.3](#_A.3). [↑](#footnote-ref-26)
27. Shepard (1974): "(...) these measures to be arrayed in the below-diagonal triangular half of an matrix which the rows and columns correspond to the same objects"*.* [↑](#footnote-ref-27)
28. For example, Castro et al (2013) use the "Non-Negative Matrix Factorization" technique to produce an olfactory similarity space, i.e. on odours*.* [↑](#footnote-ref-28)
29. Bechberger (2023, p.642): "MDS is capable of successfully compressing this information into very low-dimensional spaces". [↑](#footnote-ref-29)
30. There are several ways of calculating *Stress*. Bechberger (2023, p. 452) Borg & Groenen (2018, chap. 3) Some details in [Appendix A.5](#_A.5)*.* [↑](#footnote-ref-30)
31. Borg & Groenen (2005, p.43; 2013, p.35): "[Points] all lie on a monotonically descending line, as requested by the ordinal MDS model used to scale these data"*.* [↑](#footnote-ref-31)
32. Borg and Groenen (2018): 'Increasing the dimensionality of the MDS space always makes it easier to find a solution with a better fit'*.* If the starting data are *dissimilarity* judgments, and the MDS accurately represents these data, the points will follow a *monotonically decreasing line* in the Shepard diagram. [↑](#footnote-ref-32)
33. These judgements include similarity, categorisation and induction. This predictive performance owes much to the improvement in colour spaces, in which the nuances of colour are organised in a way that is faithful to human perception*.* [↑](#footnote-ref-33)
34. Shepard (1987, p.1319): "The data are from a number of researchers, who tested both visual and auditory stimuli, and both human and animal subjects. Yet, in every case, the decrease of generalization with psychological distance is monotonic, generally concave upward, and more or less approximates a simple exponential decay function." [↑](#footnote-ref-34)
35. Gardenfors (2000, sec. 1.6.5); Osta-Velez (2020, p.82); Bechberger (2023, p.14)*.* [↑](#footnote-ref-35)
36. Poth (2019, p.9): "For a set of stimuli, i and j , the empirical probability of an organism to generalise a type of behaviour towards j upon having observed i is a monotone and exponentially decreasing function of the distance between i and j in a continuous psychological similarity space"*.* [↑](#footnote-ref-36)
37. Borg & Groenen (2005, p.104):"(...) a cluster is a particular region whose points are all closer to each other than to any point in some other region. This makes the points in a cluster look relatively densely packed, with "empty" space around the cluster. For regions, such a requirement generally is not relevant". [↑](#footnote-ref-37)
38. Gardenfors (2000, sec. 6.6): "(...) the basic level categories of prototype theory (Rosch 1975, 1978) are characterized by distinctive clusters of correlated properties*.* [↑](#footnote-ref-38)
39. Borg & Groenen (2018, p.104): "checking to what extent items constructed to measure the same construct appear homogeneous. This analysis can be made easier by drawing convex hulls around items that belong to the same category"*.* [↑](#footnote-ref-39)
40. Henley (1969, p.180): "[the dimension] seems to be characterizable as one of mildness vs. ferocity"*.* [↑](#footnote-ref-40)
41. Henley (1969, pp.180-181): "Again there is the grouping into small animals (rabbit and mouse), herbivores (cow, deer, horse, goat, sheep) and other carnivores (bear, lion, dog, cat)"*.* [↑](#footnote-ref-41)
42. Gardenfors (2014): "These dimensions are closely connected to what is produced by our sensory receptors". [↑](#footnote-ref-42)
43. "there also exist quality dimensions that are of an abstract, non-sensory character" (2015). [↑](#footnote-ref-43)
44. Bolt et al (2019, chap.9). [↑](#footnote-ref-44)
45. Bechberger (2023, 75-78) notes the dimensions of taste by "TASTE = 〈sweet; sour; bitter; salt〉" [↑](#footnote-ref-45)